The bosonic sector of SM

The standard model of elementary particles is built on top of the Yang-Mills non-abelian gauge theory. It makes use of the nonprime gauge group — a direct product of three prime groups $SU(3) \times SU(2) \times U(1)$.

- The $SU(3)$ gauge group corresponds to the strong nuclear interaction. It has 8 gauge bosons — *gluons* — which are the massless carriers of the strong force. Through a mechanism known as confinement, all non-neutral under the $SU(3)$ charge (color) particles, such as gluons and quarks, are absent from the S-matrix at relatively low energies.
- The $SU(2) \times U(1)$ electroweak gauge group is spontaneously broken to render the gauge group of quantum electrodynamics, $U(1)_{em}$. Symmetry breaking is realized via the Higgs mechanism.
- Fermions lie in trivial or fundamental representations of the mentioned three subgroups.

The Higgs field belongs to the fundamental representation of $SU(2)$, which is complex-valued and 2-dimensional. Thus,

$$\phi(x) = \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix}.$$

Consider the Yang-Mills lagrangian with the Standard Model gauge group:

$$\mathcal{L}_{YM}[A] = \frac{1}{2e_3^2} \cdot \operatorname{tr} \underset{SU(3)}{F_{\mu\nu}^2} + \frac{1}{2e_2^2} \cdot \operatorname{tr} \underset{SU(2)}{F_{\mu\nu}^2} - \frac{1}{4} \cdot \underset{U(1)}{F_{\mu\nu}^2} +$$

$$+ D^\mu \phi^\dagger D_\mu \phi - \lambda \left(\phi^\dagger \phi - v^2\right)^2.$$

The following notation is used for the Lie algebra generators:

$$\underset{SU(2)}{A_\mu} = ie_2 \underset{SU(2)}{A_\mu^a} \frac{\sigma^a}{2}, \quad \operatorname{tr}\left[\frac{\sigma^a}{2} \cdot \frac{\sigma^b}{2}\right] = \frac{1}{2}\delta^{ab},$$

$$\underset{SU(3)}{A_\mu} = ie_3 \underset{SU(3)}{A_\mu^a} \frac{\lambda^a}{2}, \quad \operatorname{tr}\left[\frac{\lambda^a}{2} \cdot \frac{\lambda^b}{2}\right] = \frac{1}{2}\delta^{ab},$$

where $\{\sigma^1, \sigma^2, \sigma^3\}$ are Pauli matrices and $\{\lambda^1, \ldots, \lambda^8\}$ are Gell-Mann matrices. The covariant derivative of the field is determined by the choice of representations of the three gauge groups to which the considered field belongs. For the Higgs field,

$$D_\mu \phi = \partial_\mu \phi + \underset{SU(2)}{A_\mu} \phi + \frac{i}{2} e_1 \underset{U(1)}{A_\mu} \phi,$$

where the $SU(2)$ group acts on ϕ in the fundamental representation (by means of Pauli matrices) and the quantity $Y = 1/2$ determines the $U(1)$ charge of the Higgs field called the *hypercharge*.

The $U(1)$ group acts on various hypercharged fields by means of an abelian version of the gauge transformation formula:

$$\begin{cases} \phi(x) \to e^{iY_\phi e_1 \alpha_1(x)} \phi(x) \\ \underset{U(1)}{A_\mu}(x) \to \underset{U(1)}{A_\mu}(x) - \partial_\mu \alpha_1(x) \end{cases}$$

We make the standard choice for the vacuum expectation of the Higgs

$$\phi_0(x) = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

and seek a subgroup of $SU(3) \times SU(2) \times U(1)$ (the *little group*) which leaves this choice invariant. Since $SU(3)$ acts on ϕ trivially (the Higgs field is color-neutral), we choose a general anzatz for the infinitesimal $SU(2) \times U(1)$ transformation. Under the transformations from the little group, ϕ_0 by definition remains invariant:

$$\phi(x) \to e^{ie_2 \alpha_2^a(x) \frac{\sigma^a}{2}} e^{\frac{i}{2} e_1 \alpha_1(x)} \phi(x),$$

$$\delta\phi_0 = \frac{i}{2}(e_2 \alpha_2^a \sigma^a + e_1 \alpha_1)\phi_0 = 0,$$

$$\begin{pmatrix} e_1\alpha_1 + e_2\alpha_2^3 & e_2(\alpha_2^1 - i\alpha_2^2) \\ e_2(\alpha_2^1 + i\alpha_2^2) & e_1\alpha_1 - e_2\alpha_2^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Thus the little group is $U(1)_{em}$ and is generated by

$$\alpha_2^3 = \frac{e_1}{e_2}\alpha_1, \ \alpha_2^1 = \alpha_2^2 = 0.$$

This is how the electroweak model breaks down to electrodynamics. Note that the number of degrees of freedom is the same in the original version of the theory and in its low-energy theory with a broken gauge symmetry:

- In unbroken SM we have $\dim\{SU(3)\times SU(2)\times U(1)\} = 8+3+1 = 12$ massless gauge bosons (each has two polarizations giving 24 degrees of freedom total) and the Higgs complex-valued doublet. The total number of degrees of freedom is 28.
- After symmetry breaking we still have $8\cdot 2 = 16$ degrees of freedom coming from massless gluons, $3\cdot 3 = 9$ degrees of freedom coming from massive weak bosons $W^{\pm}$ and Z^0, 2 degrees of freedom coming from the massless photon and one degree of freedom representing the scalar neutral Higgs particle. The total number of degrees of freedom is still 28.

Consider now an excitation of the Higgs field around the vacuum:

$$\phi(x) = \begin{pmatrix} \varphi_1(x) + i\varphi_2(x) \\ v + \varphi_3(x) + i\varphi_4(x) \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix} + \begin{pmatrix} \varphi_1(x) + i\varphi_2(x) \\ \varphi_3(x) + i\varphi_4(x) \end{pmatrix}.$$

Under an arbitrary infinitesimal $SU(2)\times U(1)$ transformation,

$$\delta\phi(x) = \begin{pmatrix} 0 \\ v \end{pmatrix} + \begin{pmatrix} \varphi_1 + i\varphi_2 + \frac{i}{2}e_2(\alpha_2^1 - i\alpha_2^2)v \\ \varphi_3 + i\varphi_4 + \frac{i}{2}(e_1\alpha_1 - e_2\alpha_2^3)v \end{pmatrix}.$$

We see that y means of gauge transformations we can pass to the *unitary gauge*:

$$\varphi_1 = \varphi_2 = \varphi_4 = 0,$$
$$\phi = \begin{pmatrix} 0 \\ v + \varphi_3 \end{pmatrix}.$$

The quadratic approximation of the Standard Model lagrangian gives

$$\mathcal{L}_{\text{free}} = \frac{1}{2e_3}\cdot \operatorname{tr}\left(\partial_\mu \underset{SU(3)}{A_\nu} - \partial_\nu \underset{SU(3)}{A_\mu}\right)^2 - \frac{1}{2e_2}\cdot \operatorname{tr}\left(\partial_\mu \underset{SU(2)}{A_\nu} - \partial_\nu \underset{SU(2)}{A_\mu}\right)^2 -$$
$$-\frac{1}{4}\left(\partial_\mu \underset{U(1)}{A_\nu} - \partial_\nu \underset{U(1)}{A_\mu}\right)^2 + (\partial_\mu\varphi)^2 - 4\lambda v^2\varphi^2 +$$
$$+\frac{1}{4}v^2\left(e_1 \underset{U(1)}{A_\mu} - e_2 \underset{SU(2)}{A_\mu^3}\right)^2 + \frac{1}{4}e_2^2v^2\left(\underset{SU(2)}{A_\mu^1}\right)^2 + \frac{1}{4}e_2^2v^2\left(\underset{SU(2)}{A_\mu^2}\right)^2.$$

We now introduce the new variables:

$$e = e_1\cos\theta_W = e_2\sin\theta_W,$$
$$W_\mu^{\pm}(x) = \underset{SU(2)}{A_\mu^{1,2}}(x),$$
$$Z_\mu(x) = -\sin\theta_W\cdot \underset{U(1)}{A_\mu}(x) + \cos\theta_W\cdot \underset{SU(2)}{A_\mu^3}(x),$$
$$A_\mu(x) = \cos\theta_W\cdot \underset{U(1)}{A_\mu}(x) + \sin\theta_W\cdot \underset{SU(2)}{A_\mu^3}(x).$$

The quadratic approximation of the lagrangian in the new variables is

$$\mathcal{L}_{\text{free}} = \frac{1}{2e_3}\cdot \operatorname{tr}\left(\partial_\mu \underset{SU(3)}{A_\nu} - \partial_\nu \underset{SU(3)}{A_\mu}\right)^2 - \frac{1}{4}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)^2 -$$
$$-\frac{1}{4}\left(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-\right)^2 - \frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)^2 - \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 +$$
$$+\frac{1}{4}e_2^2v^2\left(W_\mu^+\right)^2 + \frac{1}{4}e_2^2v^2\left(W_\mu^-\right)^2 + \frac{1}{4}v^2\left(e_1^2 + e_2^2\right)Z_\mu^2 + (\partial_\mu\varphi)^2 - 4\lambda v^2\varphi^2.$$

This gives the full particle spectrum of the Standard Model:

- Massive charged $W^{\pm}$ bosons (the charge is determined by the way fields transform under $U(1)_{em}$) with mass $m_W = e_2 v/\sqrt{2}$.
- Massive neutral Z boson with mass $m_Z = v\sqrt{(e_1^2 + e_2^2)/2}$.
- Massless photon.
- Massive scalar Higgs particle with mass $m_\varphi = 2v\sqrt{\lambda}$.

The mass ratio

$$\frac{m_W}{m_Z} = \frac{e_2}{\sqrt{e_1^2 + e_2^2}} = \cos\theta_W$$

is called the *Weinberg angle.*

Experimental values of the Standard Model parameters. Masses of the Standard Model bosons:

$$m_\varphi \approx 125\, GeV,$$

$$m_Z \approx 91.187\, GeV,$$

$$m_W \approx 80.41\, GeV.$$

Renormalizing couplings are taken at the renormalization scale $\mu = m_Z$:

$$\alpha_3 = \frac{e_3^2}{4\pi} \approx 0.119,$$

$$\alpha^{-1} = \left(\frac{e^2}{4\pi}\right)^{-1} \approx 127.9,$$

$$\sin^2\theta_W \approx 0.2312.$$

All values are given in the $\overline{MS}$ renormalization scheme.