Bekenstein-Hawking entropy from LQG

Consider an event horizon of the black hole. In the context of LQG, the area of the event horizon is given by a sum of contributions from spin network links passing through:

$$A = 8\pi G\hbar\gamma \sum_i \sqrt{j_i\,(j_i+1)}.$$

The entropy of the black hole is by definition proportional to the logarithm of the microstates count:

$$S = k\log\Gamma.$$

Consider an observer outside the black hole. According to the holographic principle, the information about the black hole available to observer is contained in the loose ends of the spin network links. We can say that the Hilbert space of microstates which an observer assigns to the black hole is a product of irreps:

$$H = \bigotimes_i H_{j_i}.$$

We assume that the majority of microstates correspond to the situation when all N spins are equal to $1/2$ (which can be proven by a direct computation). The area is then

$$A = 4\sqrt{3}\pi G\hbar\gamma \cdot N$$

and the holographic Hilbert space is just

$$H = \left(H_{1/2}\right)^N, \quad \Gamma = \dim H = 2^N.$$

We see that entropy is proportional to the area of the event horizon:

$$\Gamma = 2^N = 2^{A/4\sqrt{3}\pi G\hbar\gamma} = \exp\left\{\frac{A\cdot\log 2}{4\sqrt{3}\pi G\hbar\gamma}\right\},$$

$$S = k\log\Gamma = \frac{k\log 2\cdot A}{4\sqrt{3}\pi G\hbar\gamma}.$$

This equals to the Bekenstein-Hawking entropy of the black hole

$$S = \frac{1}{4}\cdot\frac{kA}{G\hbar}$$

when the Immirzi parameter is set to

$$\gamma = \frac{\log 2}{\sqrt{3}\pi}.$$

In general, this is how holographic principle presents itself in LQG. The amount of information about the system available to the observer is proportional to the area of the "screen" — the boundary of the system. In case of black holes nothing comes outside the event horizon and this information is equal to the black hole entropy.